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Nonlinear Love waves in a ferroelastic film on an elastic substrate

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Abstract. A theory of nonlinear acoustic Love wave propagation in a ferroelastic film on an elastic substrate is presented. The main stress in this work is put upon the investigation of the properties of the nonlinear acoustic waves in films close to the phase transition, which is different from previously reported works on this subject. The parameters of the striction superstructure (elastic domains) nucleating close to the paraelastic–ferroelastic phase transition are estimated for real ferroelectric films. In the framework of the asymptotic approach the effective nonlinear Schrödinger equation for the envelope of the Love wave is derived and the possibility of bright- and dark-soliton formation is discussed. It is demonstrated that for a change of career wave wavenumber the transition between stable wave propagation and the modulation instability is realized. It is found that the strong dispersion of the soft Love wave near the ferroelastic phase transition is responsible for such a transition.

1. Introduction

Ferroelastic materials are interesting objects for study of different physical effects related to propagation of acoustic waves in such materials. The anomalous decrease in the values of the ferromagnetic elastic moduli close to the phase transition leads to the change of the character of the surface acoustic wave propagation. In particular, the essential decrease in Rayleigh wave velocity and, as a consequence, the increase in the penetration depth of this wave into the crystal close to the ferroelastic phase transition was predicted and calculated [1]. The polarization of this wave becomes almost transverse. These predicted features of the propagated Rayleigh wave were then experimentally observed in uniaxial ferroelectric–ferroelastic crystals [2–4].

Another interesting type of acoustic wave for practical application is the so-called Love wave. This is a pure shear wave propagating in a film–substrate elastic structure. The propagation of linear Love waves with horizontal transverse polarization in a ferroelastic film on a massive substrate was studied [5, 6]. It was shown, that for temperature $T = T_k < T_C$ (T_C is the Curie temperature of the bulk ferroelectric medium) the Love wave becomes unstable. The wave frequency and group velocity vanish for the wavenumber $k = k_c \neq 0$, and the wave condenses into the domain phase localized in the film and substrate close to their interface. Domain structure nucleation is connected to the necessary decrease in the energy of the long-range fields of the elastic stresses penetrating into the substrate interior at a distance of the order of the domain period. The sign of the dispersion coefficient for the

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Love wave $D = \partial^2\omega/\partial k^2$ depends on the wavenumber k (ω is the wave frequency). In the long-wavelength approximation ($kL \gg 1$) dispersion is negative, $D < 0$, but for $kL \ll 1$ dispersion is positive ($D > 0$) (L is the film thickness). The remarkable feature of the Love wave spectrum close to the ferroelastic phase transition is the high sensitivity to temperature variations. Elastic domain nucleation in ferroelastic–ferroelectric epitaxial films of $PbTiO_3$ grown by metal organic chemical vapour deposition on MgO(001), $SrTiO_3$, and PZT, and PZT/YBCO/$LaAlO_3$ close to the ferroelastic phase transition was experimentally observed and investigated [7–9]. Instabilities of other surface acoustic waves (e.g. Rayleigh waves) were also investigated. In particular, instability of the surface acoustic waves localized close to the plane defect was theoretically studied [10].

In the last few years the nonlinear properties of the surface acoustic waves have been intensively investigated (see for reviews [11–14]). The density of the elastic energy close to the crystal surface can be quite high due to the excitation of the acoustic waves near this surface. This, in turn, can stimulate large amplitudes of the elastic displacements at the surface. Thus the nonlinear effects become essentially important for the surface acoustic waves and can be easily observed [15–20]. Nonlinearity can cause changes in wave parameters, generation of second and higher harmonics of the surface waves [14, 16] and plate modes [17], surface wave soliton formation [21], formation of envelope solitons [11–14, 22–25] and bistability of surface waves [26, 27].

In this paper we investigate the propagation of the nonlinear shear Love wave in a ferroelastic film on an elastic substrate. We shall demonstrate that for a wavenumber change the transition between the stable wave propagation and the modulation instability is realized due to change of the wave dispersion. We shall examine the different envelope solitons for the nonlinear wave produced by the linear Love modes and show that the direction of the wave propagation becomes inverse in the region where the group velocity changes its sign.

2. Model

We shall investigate a ferroelastic film with thickness L on a semi-infinite elastic substrate occupying the space $y < 0$ (see figure 1). Let us suppose for the sake of definiteness a phase transition of symmetry D_{2h}–C_{2h} existing at the temperature T_C in the ferroelastic medium. At this transition the shear deformations η_{xz} appear spontaneously. Such transitions are typical for $KH_3(SeO_3)_2$ [28] and LaP_5O_{14} [29] crystals. The free energy of the system can be represented in the following form [14]

$$F = \sum_{n=1,2} \int_{V_n} \mathrm{d}v_n \left[\frac{1}{2} \Lambda^{(n)}_{\alpha\beta\mu\nu\gamma\delta} \frac{\partial \eta^{(n)}_{\alpha\beta}}{\partial x_\gamma} \frac{\partial \eta^{(n)}_{\mu\nu}}{\partial x_\delta} + \frac{1}{2} c^{(n)}_{\alpha\beta\mu\nu} \eta^{(n)}_{\alpha\beta} \eta^{(n)}_{\mu\nu} + \frac{1}{3!} c^{(n)}_{\alpha\beta\mu\nu\gamma\delta} \eta^{(n)}_{\alpha\beta} \eta^{(n)}_{\mu\nu} \eta^{(n)}_{\gamma\delta} \right.$$
$$\left. + \frac{1}{4!} c^{(n)}_{\alpha\beta\mu\nu\gamma\delta\sigma\lambda} \eta^{(n)}_{\alpha\beta} \eta^{(n)}_{\mu\nu} \eta^{(n)}_{\gamma\delta} \eta^{(n)}_{\sigma\lambda} + \cdots \right] \tag{1}$$

$$\eta^{(n)}_{\alpha\beta} = \tfrac{1}{2}(e^{(n)}_{\alpha\beta} + e^{(n)}_{\beta\alpha} + e^{(n)}_{\mu\alpha} + e^{(n)}_{\mu\beta}). \tag{2}$$

Here $\eta^{(n)}_{\alpha\beta}$ is the deformation tensor, $e^{(n)}_{\alpha\beta} = \partial u^{(n)}_\alpha / \partial x_\beta$ is the distortion tensor and $u^{(n)}$ is the displacement vector, $c^{(n)}_{\alpha\beta\mu\nu}$, $c^{(n)}_{\alpha\beta\mu\nu\gamma\delta}$, $c^{(n)}_{\alpha\beta\mu\nu\gamma\delta\sigma\lambda}$ are the elastic moduli of the second, third and fourth order, respectively, $\Lambda^{(n)}$ is the tensor describing the dispersion of the elastic moduli. The indices $n = 1, 2$ denote the parameters of the film and substrate respectively. The volume of the film and substrate and described by $V_{1,2}$. The components of the tensor $\Lambda^{(n)}_{\alpha\beta\mu\nu\gamma\delta}$ are proportional to $\xi_0^2 \alpha T_C$ [30] and ξ_0 is the correlation length of fluctuations far from the phase transition temperature T_C of the bulk ferroelastic material, $\xi_0 \sim b$ where b

is the lattice constant, α is a numerical constant with the dimension of the value of stress over temperature. Further we shall introduce the Voigt notation for elastic moduli and for the crystal under consideration we assume that the following moduli do not vanish

$$c_{ij}^{(n)}, c_{\alpha\alpha}^{(n)}, c_{\alpha\alpha i}^{(n)}, c_{ijk}^{(n)}, c_{\alpha\beta\gamma}^{(n)} (\alpha \neq \beta \neq \gamma), c_{ijkl}^{(n)}, c_{ij\alpha\alpha}^{(n)}, c_{i\alpha\beta\gamma}^{(n)} (\alpha \neq \beta \neq \gamma). \tag{3}$$

The indices $i, j, k, \alpha, \beta, \gamma$ will be used to describe elastic moduli in the Voigt notation: the indices i, j, k can be equal to 1, 2, 3 and the indices α, β, γ can be equal to 4, 5, 6. We assume also that close to the Curie temperature the modulus $c_{55}^{(1)} = \alpha(T - T_C)$ and the other coefficients are weakly dependent on the temperature. Furthermore, it is convenient to expand the expression for free energy in terms of the distortion tensor [31]

$$F = \sum_{n=1,2} \int_{V_n} \mathrm{d}v_n \left[\frac{1}{2} \Lambda_{\alpha\beta\mu\nu\gamma\delta}^{(n)} \frac{\partial e_{\alpha\beta}^{(n)}}{\partial x_\gamma} \frac{\partial e_{\mu\nu}^{(n)}}{\partial x_\delta} + \frac{1}{2} c_{\alpha\beta\mu\nu}^{(n)} e_{\alpha\beta}^{(n)} e_{\mu\nu}^{(n)} + \frac{1}{3!} S_{\alpha\beta\mu\eta\gamma\delta}^{(n)} e_{\alpha\beta}^{(n)} e_{\mu\nu}^{(n)} e_{\gamma\delta}^{(n)} \right.$$
$$\left. + \frac{1}{4!} S_{\alpha\beta\mu\nu\gamma\delta\sigma\lambda}^{(n)} e_{\alpha\beta}^{(n)} e_{\mu\nu}^{(n)} e_{\gamma\delta}^{(n)} e_{\sigma\lambda}^{(n)} + \cdots \right] \tag{4}$$

where the tensors S are related to the tensors c by substitution of equation (2) into (1) and manipulating the resulting expansion into the form (4). We need these conversion expressions for the first-, second- and third-order tensors [14]:

$$S_{\alpha\beta\mu\nu\gamma\delta} = c_{\alpha\beta\mu\nu\gamma\delta} + \delta_{\alpha\mu} c_{\beta\nu\gamma\delta} + \delta_{\alpha\gamma} c_{\mu\nu\beta\delta} + \delta_{\mu\gamma} c_{\alpha\beta\nu\delta} \tag{5a}$$
$$S_{\alpha\beta\mu\nu\gamma\delta\sigma\lambda} = c_{\alpha\beta\mu\nu\gamma\delta\sigma\lambda} + \delta_{\alpha\mu} c_{\sigma\lambda\gamma\delta\nu\beta} + \delta_{\alpha\gamma} c_{\sigma\lambda\mu\nu\beta\delta} + \delta_{\gamma\mu} c_{\sigma\lambda\alpha\beta\delta\nu} + \delta_{\sigma\alpha} c_{\gamma\delta\mu\nu\lambda\beta} + \delta_{\sigma\mu} c_{\gamma\delta\alpha\beta\lambda\nu}$$
$$+ \delta_{\sigma\gamma} c_{\mu\nu\alpha\beta\lambda\delta} + \delta_{\sigma\mu} \delta_{\gamma\alpha} c_{\lambda\nu\beta\delta} + \delta_{\gamma\mu} \delta_{\sigma\alpha} c_{\lambda\beta\nu\delta} + \delta_{\sigma\gamma} \delta_{\alpha\mu} c_{\lambda\delta\beta\nu} \tag{5b}$$

with $\delta_{\alpha\beta}$ the Kronecker delta. The tensors S are in general not symmetric with respect to interchanges of the two indices in a pair, in contrast to the elastic moduli.

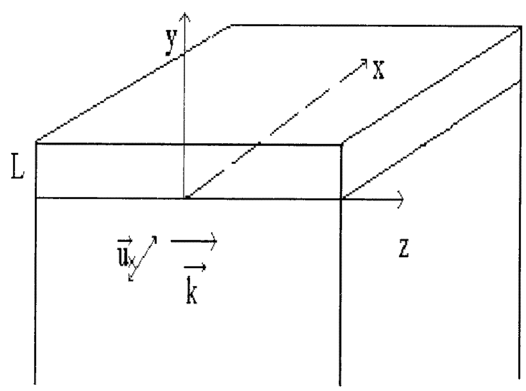

Figure 1. Geometry of the problem. The Love wave propagates along the z-axis; elastic displacement $\boldsymbol{u}$ in the wave is along the x-axis; L is the film thickness.

3. Linear Love waves

Let us consider a Love wave with the horizontal polarization $u_x^{(1)}$ propagating along the z-axis (see figure 1) in the crystal in a paraelastic phase ($T > T_k$). The elastic displacement in a Love wave can be found from the equation of motion along with the boundary conditions

$$\rho^{(n)} \frac{\partial^2 u_i^{(n)}}{\partial t^2} = \nabla_j \sigma_{ij}^{(n)} \tag{6}$$

$$\begin{aligned} &\sigma_{iy}^{(n)} = 0|_{y=L} \\ &\sigma_{iy}^{(1)} = \sigma_{iy}^{(2)} \qquad u_i^{(1)} = u_i^{(2)}(i, j = x, y, z)|_{y=0}. \end{aligned} \tag{7}$$

This displacement is

$$\begin{aligned} u_x^{(1)} &= u_0 \frac{\cos q_1(L-y)}{\cos q_1 L} \exp[\mathrm{i}(\omega t - kz)] \\ u_x^{(2)} &= u_0 \exp[\mathrm{i}(\omega t - kz) + q_2 y]. \end{aligned} \tag{8}$$

Here $\sigma_{ij}^{(n)} = \delta F/\delta e_{ij}^{(n)} - (\partial/\partial x_k)\delta F/\delta((\partial/\partial x_k)e_{ij}^{(n)})$ is the stress tensor, $\rho^{(n)}$ is the density of the ferroelastic medium,

$$\begin{aligned} q_1 &= \left\{ \frac{1}{c_{66}^{(1)}} [\rho^{(1)}\omega^2 - (\Lambda_{551}^{(1)} k^2 + c_{55}^{(1)})k^2] \right\}^{\frac{1}{2}} \\ q_2 &= k \left(\frac{c_{55}^{(2)}}{c_{66}^{(2)}} - \frac{\rho^{(2)}\omega^2}{k^2 c_{66}^{(2)}} \right)^{\frac{1}{2}}. \end{aligned} \tag{9}$$

For thick ferroelastic films $L \gg b$ the spatial dispersion of the soft elastic modulus $c_{55}^{(1)}$ influences essentially the Love wave spectrum close to the phase transition and for wavenumber such that $k \gg L^{-1}$. Thus, this fact should be taken into account for calculation of the boundaries of the stable paraelastic phase and the period of the nucleating domain structure [5]. For the indicated range of wavenumbers the Love wave period (that is proportional to its penetration depth into the interior of the substrate) is much less than the inhomogeneity of the elastic strain fields across the film, therefore the dispersion of the elastic modulus $c_{66}^{(1)}$ and the elastic moduli of the substrate can be neglected, since their influence on the Love wave spectrum is negligibly small.

The dispersion equation for Love waves has the following form

$$\operatorname{cotan}(q_1 L) = \frac{q_1 c_{66}^{(1)}}{q_2 c_{66}^{(2)}}. \tag{10}$$

Equation (10) has an analytical solution in two limiting cases. For $kL \ll 1$ the dispersion equation is [13]

$$\omega^2 = \omega_k^2 = c_{55}^{(2)} k^2 \left[1 - \frac{\rho^{(1)^2}}{\rho^{(2)^2}} \frac{c_{55}^{(2)}}{c_{66}^{(2)}} k^2 L^2 \right] \Big/ \rho^{(1)}. \tag{11}$$

For $kL \gg 1$ the dispersion equation can be represented as [5]

$$\omega^2 = \omega_{mk}^2 = [(c_{55}^{(1)} + \Lambda_{551}^{(1)} k^2)k^2 + (2m-1)^2 \pi^2 c_{66}^{(1)}/4L^2]/\rho^{(1)} \qquad m = 1, 2, \ldots. \tag{12}$$

It should be noted that each new solution (new surface Love wave) emerges at $k = k_m = \pi m/L$. The mode with $m = 1$ is the soft mode. For this mode one can write

$$\begin{aligned} q_1 &\approx \pi[1 - (kL)^{-1}(c_{55}^{(2)} c_{66}^{(2)})^{-\frac{1}{2}} c_{66}^{(1)}]/2L \qquad (q_1 \ll k) \\ q_2 &\approx k(c_{55}^{(2)}/c_{66}^{(2)})^{-\frac{1}{2}}. \end{aligned} \tag{13}$$

The spectrum of the soft Love wave is shown in figure 2. The temperature of the loss of stability of the paraelastic phase with respect to the nucleation of the domain structure T_k

and the wavenumber k_c of the critical domain structure can be calculated from the condition of the vanishing of the Love wave frequency and the group velocity

$$\omega_{1k}^2 = 0 \qquad \frac{\mathrm{d}\omega_{1k}^2}{\mathrm{d}k} = 0. \tag{14}$$

They are

$$T_k = T_C - \pi\alpha^{-1}L^{-1}(\Lambda_{551}^{(1)}c_{66}^{(1)})^{1/2}$$
$$k_c = 2\pi/d_c = \left(\frac{\pi^2}{4L^2}\frac{c_{66}^{(1)}}{\Lambda_{551}^{(1)}}\right)^{\frac{1}{4}}. \tag{15}$$

Here d_c is the period of the critical domain structure. It follows from (15) that the phase transition in the film appears at a temperature which is less than the phase transition temperature for the bulk ferroelastic medium ($T_k < T_c$). Estimations show that for the $KH_2(SeO_2)_3$ crystal [28]

$$d_c \sim 1.4 \times 10^{-3}L^{1/2}\ \text{cm} \qquad T_C - T_k \sim 3.34 \times 10^{-5}L^{-1}\ \text{K}$$

L is expressed in centimetres.

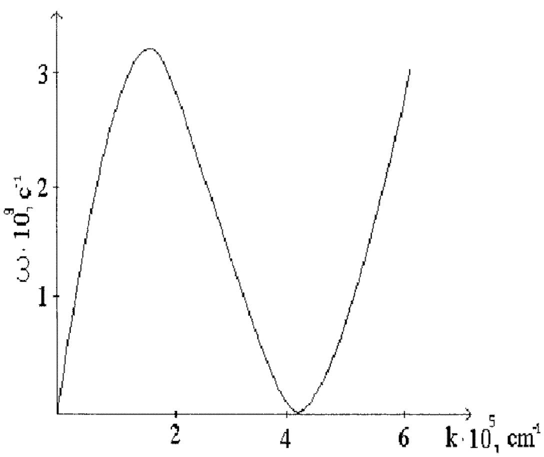

Figure 2. The spectrum of the critical Love wave at $T = T_C$ in a film of $KH_3(SeO_3)_2$ on a fused quartz substrate ($L = 10^{-4}$ cm).

For thick ferroelastic films ($L \gg b$) the following inequalities are valid:

$$q_1 \ll k_c \ll b^{-1} \qquad d_c \ll L$$

and the penetration depth of the surface solution into the substrate is

$$l = q_2^{-1} \sim d_c \ll L.$$

For $k \sim k_c$ and $T \sim T_k$ equation (12), describing the dispersion relation for the critical Love wave, can be represented in the following form

$$\omega^2 = \omega_{1k}^2 = [\alpha^2(T - T_k)(T_C - T_k)/(2\Lambda_{551}^{(1)}) + \Lambda_{551}^{(1)}(k^2 - k_c^2)^2]/\rho^{(1)}. \tag{16}$$

The spectrum of the critical Love wave for $T \sim T_k$ in the $KH_3(SeO_3)_2$ film is shown in figure 2.

4. Nonlinear Love waves. Envelope solitons

In case of nonlinear Love waves we search for the solution to the set of equations (6) with the boundary conditions (7) in the following form

$$
\begin{aligned}
u_x^{(n)} &= \sum_{p=0}^{\infty} A_{2p+1}^{(n)}(y)\cos(2p+1)\theta \\
u_y^{(n)} &= \sum_{p=0}^{\infty} B_{2p}^{(n)}(y)\cos 2p\theta \\
u_z^{(n)} &= \sum_{p=1}^{\infty} D_{2p}^{(n)}(y)\sin 2p\theta.
\end{aligned} \tag{17}
$$

Here $\theta = kz - \omega t$, $n = 1, 2$ and functions A, B, D should be expanded in formal asymptotic series

$$
\begin{aligned}
A_{2p+1}^{(n)} &= \sum_{m=0}^{\infty} \varepsilon^{2p+2m+1} A_{2p+1,2m+1}^{(n)} \\
B_{2p}^{(n)} &= \sum_{m=0}^{\infty} \varepsilon^{2(m+p)} B_{2p,2m}^{(n)} \text{ for } p \neq 0 \qquad B_0^{(n)} = \sum_{m=0}^{\infty} \varepsilon^{2(m+1)} B_{0,2m}^{(n)} \\
D_{2p}^{(n)} &= \sum_{m=0}^{\infty} \varepsilon^{2(m+p)} D_{2p,2m}^{(n)}
\end{aligned} \tag{18}
$$

where ε is a small parameter proportional to the Love wave amplitude.

Substituting equation (17) into equations (6) and (7) and equating to zero coefficients at different harmonics we obtain the infinite set of differential equations with the boundary conditions. In the first-order approximation on ε the solutions to these equations are given as

$$
\begin{aligned}
A_1^{(1)} &= a_1 \cos q_1 y + a_1^* \sin q_1 y \\
A_1^{(2)} &= f_1 \exp q_2 y.
\end{aligned} \tag{19}
$$

Further we shall consider in detail the nonlinear Love waves produced by the lowest (soft) linear mode ($m = 1$). In this case we have

$$
a_1 = f_1 \approx \frac{a_1^*}{q_1 L} \gg a_1^* \qquad q_1 \approx \sqrt{q_2 c_{66}^{(2)}/(L c_{66}^{(1)})} \qquad q_2 \approx \sqrt{(c_{55}^{(2)} k^2 - \rho^{(2)}\omega^2)/c_{66}^{(2)}}
$$

in the long-wavelength limit $kL \ll 1$ and

$$
a_1 = f_1 \approx a_1^* \operatorname{cotan} q_1 L \ll a_1^* \qquad q_1 \approx \pi/2L \ll k_c \qquad q_2 \approx k\sqrt{c_{55}^{(2)}/c_{66}^{(2)}}
$$

for $k \sim k_c$; a_1 and a_1^* are both real. The dispersion equation for the nonlinear Love wave is

$$
\omega_{NL} = \omega_k + \varepsilon^2 N(a_1^2 + a_1^{*2}) \tag{20}
$$

and ω_k is described by formulae (11) and (16) in the liner approximation for $kL \ll 1$ and $k \sim k_c$, respectively, N is called the nonlinear term and is described as $N = \frac{3}{16}(k^6 L^2/\rho^{(2)}\omega_k) c_{55}^{(2)}/c_{66}^{(2)} S_1^{(1)*}$,

$$
S_1^{(1)*} = S_1^{(1)} - \frac{4}{9}\frac{\rho^{(1)}}{\rho^{(2)}}\frac{c_{55}^{(2)}}{c_{66}^{(1)}} S_3^{(1)} + \left(\frac{\rho^{(1)}}{\rho^{(2)}}\frac{c_{55}^{(2)}}{c_{66}^{(1)}}\right)^2 S_2^{(1)} \tag{21}
$$

for $kL \ll 1$ and

$$N = \frac{3k^4 S_1^{(1)*}}{64\rho^{(1)}\omega_{1k}} \qquad S_1^{(1)*} = S_1^{(1)} - \frac{T_1^{(1)^2}}{c_{33}^{(1)}} - \frac{2T_2^{(1)^2}}{c_{22}^{(1)}} \tag{22}$$

for $k \sim k_c$. The following definitions were used while deriving equations (21) and (22)

$$S_1^{(1)} \equiv S_{xzxzxzxz}^{(1)} = c_{5555}^{(1)} + 6c_{355}^{(1)}$$
$$S_2^{(1)} \equiv S_{xyxyxyxy}^{(1)} = c_{6666}^{(1)} + 6c_{266}^{(1)}$$
$$S_3^{(1)} \equiv S_{xyxyxzxz}^{(1)} = c_{5566}^{(1)} + c_{255}^{(1)} + c_{366}^{(1)} + 4c_{456}^{(1)}$$
$$T_1^{(1)} \equiv S_{xzxzzz}^{(1)} = c_{355}^{(1)} + c_{33}^{(1)}$$
$$T_2^{(1)} \equiv S_{xzxzyy}^{(1)} = c_{255}^{(1)} + c_{23}^{(1)}.$$

The detailed derivation of the spectrum of the nonlinear Love wave is presented in the appendix. We use results obtained in the appendix to describe the properties of the nonlinear waves and envelope solitons. According to the Lighthill criterion [32] a nonlinear wave with a stationary profile becomes modulationally unstable if the following inequality holds:

$$D/N < 0. \tag{23}$$

In the long-wavelength approximation ($kL \ll 1$) the Love wave possesses negative dispersion

$$D = -\frac{3k^4 L^2}{\rho^{(2)^2}\omega_k^3} \frac{c_{55}^{(2)^3}}{c_{66}^{(2)}} < 0. \tag{24}$$

Therefore this wave is modulationally stable for $S_1^{(1)*} < 0$ and unstable for $S_1^{(1)*} > 0$. Since the sign of the dispersion coefficient depends on the wavenumber of the wave, it is possible to see the transition from the stable regime of propagation to the unstable one simple by changing the wavenumber or the wave frequency. For $k \sim k_c$ the wave dispersion is positive, $D = 4k_c^2\Lambda_{551}^{(1)}/(\rho^{(1)}\omega_{1k}) > 0$. Therefore, for this case, the nonlinear Love wave is modulationally stable for $S_1^{(1)*} > 0$ and is modulationally unstable for $S_1^{(1)*} < 0$. The case when $S_1^{(1)*} > 0$ is realized when the ferroelastic phase transition is the phase transition of the second order, and the case when $S_1^{(1)*} < 0$ is typical for the ferroelastic phase transition of the first order [5]. The modulation instability can lead to the appearance of a set of solitons propagating along the direction of the group velocity of the initial wave.

From the dispersion equations for nonlinear Love waves (20) one can easily obtain the nonlinear Schrödinger equation for the envelope amplitude of the wave using the methods of geometrical optics [33]. The complex envelope amplitude of the wave is of the following form

$$A = \varepsilon a \exp(\mathrm{i}\theta) + \mathrm{CC}. \tag{25}$$

We denote also $\theta = kz - \omega t + \varphi$, $\varphi =$ constant, and $a \equiv a_1^*$ for $kL \ll 1$ and $a \equiv a_1$ for $k \sim k_c$. Assuming that a and φ are functions slowly varying in time and space, the frequency ω and wavenumber k can be defined in the following manner

$$\omega(z,t) = -\frac{\partial\theta}{\partial t} = \omega(k = k_0) - \frac{\partial\varphi}{\partial t} \qquad k(z,t) = \frac{\partial\theta}{\partial z} = k_0 + \frac{\partial\varphi}{\partial z} \tag{26}$$

where k_0 is the wavenumber of the linear wave. Substituting equation (25) into equation (20) and expanding the function $\omega(k)$ into a series in $(k - k_0)$ close to the linear value k_0, one obtains the nonlinear equation in second-order approximation

$$\frac{\partial\varphi}{\partial t} + v_g\frac{\partial\varphi}{\partial z} + \frac{1}{2}D\left(\frac{\partial\varphi}{\partial z}\right)^2 + N\varepsilon^2 a^2 = 0 \tag{27}$$

where the Love wave group velocity is $v_g = \partial\omega/\partial k$; v_g and D are defined at $k = k_0$, $a = 0$. Combining equation (27) with the equation of continuity

$$\frac{\partial a^2}{\partial t} + \frac{\partial(a^2 v_g)}{\partial z} = 0 \tag{28}$$

we obtain the following nonlinear parabolic equation

$$\mathrm{i}\left(\frac{\partial A}{\partial t} + v_g \frac{\partial A}{\partial z}\right) + \frac{1}{2} D \frac{\partial^2 A}{\partial z^2} - \varepsilon^2 N |A|^2 A = 0 \tag{29}$$

and $A = a\exp(\mathrm{i}\varphi)$. The solution of equation (29) depends essentially on the sign of coefficients N and D. For example, this equation has the simplest localized solution in the long-wave approximation ($kL \ll 1$), when $N > 0$, and for $k \sim k_c$, when $N < 0$. It is

$$A = A_0 \operatorname{sech} \frac{(z - v_g t)}{z_0} \exp\left(\frac{\mathrm{i}t}{T}\right) \tag{30}$$

where $z_0^{-1} = \varepsilon A_0 \sqrt{|N/D|}$, $T^{-1} = -N\varepsilon^2 A_0^2/2$. This solution corresponds to the localized solution for the elastic displacement

$$\begin{aligned} u_x^{(1)} &= a(y) \operatorname{sech}\left(\frac{z - v_g t}{z_0}\right) \cos(kz - \Omega t) \\ u_x^{(2)} &= f \exp(q_2 y) \operatorname{sech}\left(\frac{z - v_g t}{z_0}\right) \cos(kz - \Omega t) \end{aligned} \tag{31}$$

where $a(y) \approx a_1 \cos q_1 y$ and $f = a_1$ for $kL \ll 1$; $a(y) \approx a_1^{(*)} \sin q_1 y$ and $f = \pi a_1^{(*)}/k_c L$ for $k \sim k_c$. The wave frequency $\Omega = \omega_k - 1/T$ is related to the wavenumber by the dispersion relations (20) in which, however, N should be replaced by $N/2$. The wave frequency Ω is higher (respectively, lower) than the frequency of the linear Love wave for $kL \ll 1$ ($k \sim k_c$). We note here that the soliton velocity v_g changes its sign at the point where $k = k_c$. The solution (31) describes the so-called bright solitons. However, equation (29) also has a solution in the form of localized perturbations of the stable plane wave (so-called dark solitons) for $kL \ll 1$ ($N < 0$) or for $k \sim k_c$ ($N > 0$). This solution is

$$\begin{aligned} u_x^{(1)} &= a(y) \tanh \frac{(z - v_g t)}{z_0} \cos(kz - \Omega t) \\ u_x^{(2)} &= f \exp(-q_2 y) \tanh \frac{(z - v_g t)}{z_0} \cos(kz - \Omega t) \end{aligned} \tag{32}$$

where the wave frequency $\Omega = \omega_k - 2/T$ is described by the relations (20) for $kL \ll 1$ and $k \sim k_c$, respectively. This frequency is lower (or higher) than the linear Love wave frequency. Dark solitons can be excited simply by change the phase of the generator [34]. It should be noted that the developed asymptotic approach makes it possible to calculate the nonlinear waves and the discrete set of solitons produced by the higher linear Love modes ($m > 1$). It is easy to show that the dispersion equations for the nonlinear Love waves are also described by equation (20) where $\omega = \omega_{mk}$ (12) with $m > 1$ and N depending on the combination between the elastic moduli and the wavenumber k. For $m > 1$ solitonic solutions of the nonlinear parabolic equation (29) are analysed in the same way as the solitons for the soft Love wave. For $k_m > k_c$ the velocities of the nonlinear wave and solitonic excitations change sign at $k = k_c$.

5. Conclusion

In the present work we considered propagation of a nonlinear Love wave in a thick ($L \gg b$) ferroelastic film situated on a massive substrate and in the case when the temperature is close to the ferroelastic phase transition. We show the existence of different types of nonlinear excitation, namely bright solitons and the waves of the stationary profile. We show also that for a change of wavenumber the direction of the soliton propagation becomes reversed. Similar features of nonlinear wave propagation are expected in magnetic and ferroelectric crystals and solid solutions close to phase transitions accompanied by nucleation of the modulated and domain structures. We note here that different phase transitions in magnetics (see, e.g., [35]) are ferroelastic phase transitions according to their symmetry classification. One can govern the parameters of nonlinear acoustic waves in such ferroelastics by external magnetic or electric fields. We plan to consider in future work the features of nonlinear acoustic waves in superthin films with thickness $L \sim b$ grown on massive substrates.

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Appendix

In this appendix we introduce the scheme for the derivation of the dispersion equation for the nonlinear Love wave in a first-order approximation on a small parameter ε. In this approximation we introduce the solution (17) of the set of equations (6) with the boundary conditions (7) as

$$u_x^{(n)} = A_1^{(n)}(y)\cos\theta \qquad u_y^{(n)} = B_0 + B_2(y)\cos 2\theta \tag{A1}$$

$$u_z^{(n)} = D_2(y)\sin 2\theta. \tag{A2}$$

Substituting equations (A1) and (A2) into equations (6) and (7) and equating to zero coefficients at different harmonics we obtain the infinite set of differential equations with the boundary conditions. They are

$$\begin{aligned}
&\left(\rho^{(n)}\omega^2 + c_{66}^{(n)}\frac{\partial^2}{\partial y^2} - c_{55}^{(n)}k^2 - \Lambda_{551}^{(n)}k^4\right)A_1^{(n)} + \frac{\varepsilon^2}{8}\left\{-S_1^{(n)}k^4 A_1^{(n)^3} + S_2^{(n)}\frac{\partial}{\partial y}\left(\frac{\partial A_1^{(n)}}{\partial y}\right)^3\right.\\
&\qquad\left. -\frac{2}{3}S_1^{(n)}k^2\left[A_1^{(n)}\left(\frac{\partial A_1^{(n)}}{\partial y}\right)^2 - \frac{\partial}{\partial y}\left(A_1^{(n)^2}\frac{\partial A_1^{(n)}}{\partial y}\right)\right]\right\}\\
&\qquad +\varepsilon^2\left\{T_4^{(n)}\frac{\partial}{\partial y}\left[\frac{\partial A_1^{(n)}}{\partial y}\frac{\partial}{\partial y}\left(B_0^{(n)} + \frac{1}{2}B_2^{(n)}\right)\right] - T_2^{(n)}k^2 A_1^{(n)}\frac{\partial}{\partial y}\left(B_0^{(n)} - \frac{1}{2}B_2^{(n)}\right)\right.\\
&\qquad +T_3^{(n)}k\frac{\partial}{\partial y}\left(D_2^{(n)}\frac{\partial A_1^{(n)}}{\partial y}\right) + T_1^{(n)}k^3 A_1^{(n)}D_2^{(n)}\\
&\qquad +\frac{1}{2}T_5^{(n)}k\left[\frac{\partial A_1^{(n)}}{\partial y}\left(-2kB_2^{(n)} + \frac{\partial D_2^{(n)}}{\partial y}\right)\right.\\
&\qquad \left.\left. -\frac{\partial}{\partial y}\left[A_1^{(n)}\left(-2kB_2^{(n)} + \frac{\partial D_2^{(n)}}{\partial y}\right)\right]\right]\right\} = 0
\end{aligned} \tag{A3}$$

$$4\left(\rho^{(n)}\omega^2+\frac{c_{22}^{(n)}}{4}\frac{\partial^2}{\partial y^2}-k^2c_{44}^{(n)}\right)B_2+2(c_{44}^{(n)}+c_{23}^{(n)})k\frac{\partial D_2^{(n)}}{\partial y}$$
$$=\frac{1}{4}T_2^{(n)}k^2\frac{\partial A_1^{(n)^2}}{\partial y}-\frac{1}{4}T_4^{(n)}\frac{\partial}{\partial y}\left(\frac{\partial A_1^{(n)}}{\partial y}\right)^2+k^2T_5^{(n)}A_1^{(n)}\frac{\partial A_1^{(n)}}{\partial y} \tag{A4}$$

$$c_{22}^{(n)}\frac{\partial B_0^{(n)}}{\partial y^2}=-\frac{1}{4}\left[T_4^{(n)}\frac{\partial}{\partial y}\left(\frac{\partial A_1^{(n)}}{\partial y}\right)^2+k^2T_2^{(n)}\frac{\partial A_1^{(n)^2}}{\partial y}\right] \tag{A5}$$

$$4\left(\rho^{(n)}\omega^2+\frac{c_{44}^{(n)}}{4}\frac{\partial^2}{\partial y^2}-k^2c_{33}^{(n)}\right)D_2^{(n)}-2k(c_{44}^{(n)}+c_{23}^{(n)})\frac{\partial B_2^{(n)}}{\partial y}$$
$$=\frac{k}{2}\left[T_3^{(n)}\left(\frac{\partial A_1^{(n)^2}}{\partial y}\right)^2-k^2T_1^{(n)}A_2^{(n)^2}+T_5^{(n)}\frac{\partial}{\partial y}\left(A_1^{(n)}\frac{\partial A_1^{(n)}}{\partial y}\right)\right]. \tag{A6}$$

Here

$$S_1^{(n)}\equiv S_{xzxzxzxz}^{(n)}=c_{5555}^{(n)}+6c_{355}^{(n)}$$
$$S_2^{(n)}\equiv S_{xyxyxyxy}^{(n)}=c_{6666}^{(n)}+6c_{266}^{(n)}$$
$$S_3^{(n)}\equiv S_{xyxyxzxz}^{(n)}=c_{5566}^{(n)}+c_{255}^{(n)}+c_{366}^{(n)}+4c_{456}^{(n)}$$
$$T_1^{(n)}\equiv S_{xzxzzz}^{(n)}=c_{355}^{(n)}+c_{33}^{(n)}$$
$$T_2^{(n)}\equiv S_{xzxzyy}^{(n)}=c_{255}^{(n)}+c_{23}^{(n)}$$
$$T_3^{(n)}\equiv S_{xyxyzz}^{(n)}=c_{366}^{(n)}+c_{23}^{(n)}$$
$$T_4^{(n)}\equiv S_{xyxyyy}^{(n)}=c_{266}^{(n)}+c_{22}^{(n)}$$
$$T_5^{(n)}\equiv S_{xyyzxz}^{(n)}=c_{456}^{(n)}+c_{44}^{(n)}.$$

The boundary conditions at the surface $y=0$ are

$$A_1^{(1)}=A_1^{(2)}\qquad B_0^{(1)}=B_0^{(2)}\qquad B_2^{(1)}=B_2^{(2)}\qquad D_2^{(1)}=D_2^{(2)} \tag{A7}$$

$$\left\{c_{66}^{(1)}+\varepsilon^2\left[\frac{1}{8}S_2^{(1)}\left(\frac{\partial A_1^{(1)}}{\partial y}\right)^2+\frac{1}{12}k^2S_3^{(1)}A_1^{(1)^2}+2kT_3^{(1)}D_2^{(1)^2}\right]\right\}\frac{\partial A_1^{(1)}}{\partial y}$$
$$+2\varepsilon^2T_4^{(1)}\frac{\partial}{\partial y}\left[\left(B_0^{(1)}+\frac{1}{2}B_2^{(1)}\right)\frac{\partial A_1^{(1)}}{\partial y}\right]+2\varepsilon^2kT_5^{(1)}A_1^{(1)}\left(2kB_2^{(1)}-\frac{\partial D_2^{(1)}}{\partial y}\right)$$
$$=c_{66}^{(2)}\frac{\partial A_1^{(2)}}{\partial y} \tag{A8}$$

$$c_{22}^{(1)}\frac{\partial B_0^{(1)}}{\partial y}+\frac{1}{4}\left[T_2^{(1)}k^2A_2^{(1)^2}+T_4^{(1)}\left(\frac{\partial A_1^{(1)}}{\partial y}\right)^2\right]=c_{22}^{(2)}\frac{\partial B_0^{(2)}}{\partial y} \tag{A9}$$

$$c_{44}^{(1)}\left(-2kB_2^{(1)}+\frac{\partial D_2^{(1)}}{\partial y}\right)-\frac{k}{2}T_5^{(1)}A_1^{(1)}\frac{\partial A_1^{(1)}}{\partial y}=c_{44}^{(2)}\left(-2kB_2^{(2)}+\frac{\partial D_2^{(2)}}{\partial y}\right) \tag{A10}$$

$$c_{22}^{(1)}\frac{\partial B_2^{(1)}}{\partial y}+2kc_{23}^{(1)}D_2^{(1)}-\frac{k^2}{4}T_2^{(1)}A_2^{(1)^2}+\frac{1}{4}T_4^{(1)}\left(\frac{\partial A_1^{(1)}}{\partial y}\right)^2=c_{22}^{(2)}\frac{\partial B_2^{(2)}}{\partial y}+2kc_{33}^{(2)}D_2^{(2)}. \tag{A11}$$

The boundary conditions at the surface $y=L$ are described by equations (A8)–(A11) where, however, the right-hand side is equal to zero. In following we consider the case when a film of nonlinear medium is located on the surface of a substrate of linear elastic medium. This saves us tedious calculations but does not violate the generality of considerations. Therefore, we do not include the nonlinear terms in the right-hand side of equations (A8)–(A11) and we shall neglect such terms in equations (A3)–(A6) for the substrate ($n = 2$).

We search for the solution to the set of equations in the following form

$$\begin{aligned}
A_1^{(1)} &= a_1 \cos q_1 y + a_1^* \sin q_1 y \\
B_0^{(1)} &= b_{00} y + b_0 \sin 2q_1 y + b_0^* \cos 2q_1 y \\
B_2^{(1)} &= \sum_{i=1,2} (b_2^{(i)} \sinh q_1^{(i)} y + b_2^{(i)^*} \cosh q_1^{(i)} y) + b_2 \sin 2q_1 y + b_2^* \cos 2q_1 y \\
D_2^{(1)} &= \sum_{i=1,2} (d_2^{(i)} \cosh q_1^{(i)} y + d_2^{(i)^*} \sinh q_1^{(i)} y) + d_2 \cos 2q_1 y + d_2^* \sin 2q_1 y \\
A_1^{(2)} &= f_1 \exp(q_2 y) \qquad B_2^{(2)} = \sum_{i=1,2} f_2^{(i)} \exp(q_2^{(i)} y) \qquad D_2^{(2)} = \sum_{i=1,2} f_2^{(i)^*} \exp(q_2^{(i)} y).
\end{aligned} \tag{A12}$$

Here the coefficients $b_2^{(i)}$, $b_2^{(i)^*}$, $d_2^{(i)}$, $d_2^{(i)^*}$, $f_2^{(i)}$ and $f_2^{(i)^*}$ are calculated from the boundary conditions (A7)–(A11), the parameters q_1 and q_2 describing the distribution of the elastic displacements inside the film and substrate are found from the following relations

$$\Omega^2(\omega, k, q_1) - \varepsilon^2 N_1 (a_1^2 + a_1^{*^2}) = 0 \tag{A13}$$

$$q_2 = \sqrt{(c_{55}^{(2)} k^2 - \rho^{(2)} \omega^2)/c_{66}^{(2)}}. \tag{A14}$$

The remaining coefficients are

$$\begin{aligned}
b_2^{(1)} &= \varepsilon^2 q_1 (a_1^2 - a_1^{*^2}) G_b / 16\Delta \qquad b_2^{(1)*} = -\varepsilon^2 q_1 a_1 a_1^* G_b / 8\Delta \\
d_2^{(1)} &= \varepsilon^2 k (a_1^2 - a_1^{*^2}) G_d / 16\Delta \qquad d_2^{(1)*} = \varepsilon^2 k a_1 a_1^* G_d / 8\Delta \\
b_0^{(1)} &= -\varepsilon^2 \gamma_2 (a_1^2 - a_1^{*^2}) / 16 q c_{22}^{(1)} \qquad b_0^{(1)*} = \varepsilon^2 \gamma_2 a_1 a_1^* / 8 q c_{22}^{(1)} \\
d_0 &= \varepsilon^2 k \gamma_3 (a_1^2 + a_1^{*^2}) / 16 (c_{33}^{(1)} k^2 \rho^{(1)} \omega^2).
\end{aligned} \tag{A15}$$

In expressions (A12)–(A15) the following notations are adopted

$$\begin{aligned}
q_n^{(1,2)} &= 2(Q_{1n} \pm \sqrt{Q_{1n}^2 - Q_{2n}})/c_{22}^{(n)} c_{44}^{(n)} \\
Q_{1n} &= k^2 [c_{22}^{(n)} c_{33}^{(n)} + c_{44}^{(n)^2} - (c_{44}^{(n)} + c_{23}^{(n)})^2] - \rho^{(n)} \omega^2 (c_{44}^{(n)} + c_{22}^{(n)}) \\
Q_{2n} &= 4 c_{22}^{(n)} c_{44}^{(n)} (\rho^{(n)} \omega^2 - k^2 c_{44}^{(n)}) (\rho^{(n)} \omega^2 - k^2 c_{33}^{(n)}) \qquad (n = 1, 2) \\
G_b &= g_2 \Omega_3 - k^2 g_3 (c_{44}^{(1)} + c_{33}^{(1)}) \qquad G_d = g_3 \Omega_2 - q_1^2 g_2 (c_{44}^{(1)} + c_{33}^{(1)}) \\
\gamma_2 &= T_2^{(1)} k^2 - T_4^{(1)} q_1^2 \qquad \gamma_3 = T_1^{(1)} k^2 - T_3^{(1)} q_1^2
\end{aligned} \tag{A16}$$

$$\Delta = c_{22}^{(1)} c_{44}^{(1)} \left(q_1^2 + \frac{q_1^{(1)^2}}{4} \right) \left(q_1^2 + \frac{q_1^{(2)^2}}{4} \right) \tag{A17}$$

$$\begin{aligned}
g_2 &= k^2 (T_2^{(1)} + 2 T_5^{(1)}) + q_1^2 T_4^{(1)} \qquad g_3 = q_1^2 (T_3^{(1)} + 2 T_5^{(1)}) + k^2 T_1^{(1)} \\
\Omega_2 &= q_1^2 c_{22}^{(1)} + k^2 c_{44}^{(1)} - \rho^{(1)} \omega^2 \qquad \Omega_3 = k^2 c_{33}^{(1)} + q_1^2 c_{44}^{(1)} - \rho^{(1)} \omega^2 \\
\Omega^2(\omega, k, q_1) &= \omega^2 - (q_1^2 c_{66}^{(1)} + k^2 c_{55}^{(1)} + k^4 \Lambda_{551}^{(1)})/\rho^{(1)}
\end{aligned} \tag{A18}$$

$$N_1 = \frac{3}{32\rho^{(1)}}\left[k^4 S_1^{(1)} + \frac{4}{9}k^2 q_1^2 S_3^{(1)} + q_1^4 S_2^{(1)} - \frac{1}{3\Delta}(k^2 g_3^2 \Omega^2 + q_1^2 g_2^2 \Omega_3 - 2k^2 q_1^2 g_2 g_3 (c_{44}^{(1)} + c_{23}^{(1)})) - \frac{2}{3}\left(\frac{k^2\gamma_3^2}{c_{33}^{(1)}k^2 - \rho^{(1)}\omega^2} + \frac{\gamma_2^2}{c_{22}^{(1)}}\right)\right]. \tag{A19}$$

To calculate the spectrum of nonlinear waves we substitute the solution (A12) into the equations for the boundary conditions (A7)–(A11). Simple analytical expressions for the spectrum of nonlinear Love waves can be calculated in two limiting cases $k \sim k_c$ and $kL \ll 1$. For $k \sim k_c$ we obtain the expressions for the amplitudes of the fundamental harmonic

$$a_1 = f_1 = a_1^* \operatorname{cotan}(q_1 L) \qquad \operatorname{cotan}(q_1 L) \approx \frac{q_1 c_{66}^{(1)}}{q_2 c_{66}^{(2)}}$$
$$q_1 \approx \frac{\pi}{2L} \ll k_c \qquad a_1^* \gg a_1 \qquad q_2 = \sqrt{(c_{55}^{(2)}k^2 - p^{(2)}\omega^2)/c_{66}^{(2)}}. \tag{A20}$$

In this case the dispersion equation for the nonlinear Love wave is

$$\omega_{NL} = \omega_{1k} + \varepsilon^2 N a_1^{*^2} \tag{A21}$$

and ω_{1k} is described by the formula (16) in the linear approximation and the following notations are used

$$N = \frac{N_1(\omega = 0, k, q_1 = 0)}{2\omega_{1k}} = \frac{3k^4 S_1^{(1)^*}}{64\rho^{(1)}\omega_{1k}} \qquad S_1^{(1)^*} = S_{(1)}^{(1)} - \frac{T_1^{(1)^2}}{c_{33}^{(1)}} - \frac{2T_2^{(1)^2}}{c_{22}^{(1)}}. \tag{A22}$$

It should be noted that the inclusion of the cubic terms in the expression for the free energy (1) leads to the appearance of the sagittal components for the displacement u_y and u_z and to the renormalization of the elastic modulus $S_1^{(1)^*}$.

For $kL \ll 1$ the amplitudes of the fundamental harmonics can be represented as

$$a_1 = f_1 \approx \frac{a_1^*}{q_1 L} \gg a_1^* \qquad q_1 \approx \sqrt{q_2 c_{66}^{(2)}/(L c_{66}^{(1)})} \qquad q_2 \approx \sqrt{(c_{55}^{(2)}k^2 - \rho^{(2)}\omega^2)/c_{66}^{(2)}}. \tag{A23}$$

The dispersion equation for the nonlinear Love wave is

$$\omega_{NL} = \omega_k + \varepsilon^2 N a_1^2 \tag{A24}$$

with

$$N = \frac{3}{16}\frac{k^6 L^2}{\rho^{(2)}}\frac{c_{55}^{(2)}}{c_{66}^{(2)}} S_1^{(1)^*} \tag{A25}$$

$$S_1^{(1)^*} = S_1^{(1)} - \frac{4}{9}\frac{\rho^{(1)}}{\rho^{(2)}}\frac{c_{55}^{(2)}}{c_{66}^{(1)}} S_3^{(1)} + \left(\frac{\rho^{(1)}}{\rho^{(2)}}\frac{c_{55}^{(2)}}{c_{66}^{(1)}}\right)^2 S_2^{(1)}. \tag{A26}$$

Deriving (A25) and (A26) we took into account that close to the ferroelastic phase transition the following inequality is valid: $|c_{55}^{(1)}| \ll c_{55}^{(2)}$. We neglected also (without the loss of generality) the additions to $S_1^{(1)^*}$ due to sagittal components of the elastic displacements while deriving (A26).

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